Indirect Taxes in Multisectoral Macroeconomic Models

A Contribution from the Inforum group

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(Some) Matrices behind a Macroeconomic Multisectoral Model

- Supply and Use matrices
- Domestic and imported flows
- Symmetric Input-output matrices
- Margin matrix
- Excise Tax Matrix
- *ad valorem* tax flows (EU VAT Matrix)
- Bridge Matrices (Consumption, Investments)
Tax Flows in IO tables

Let us consider intermediate flows in terms of:

- $q$ the real flow
- $p$ the corresponding price
- $s$ the amount of excise tax
- $t$ the rate of the *ad valorem* tax
INTERMEDIATE FLOWS AT BASIC PRICES

Let us consider an intermediate consumption element $X_{ij}$ at basic prices

$$X_{ij} = q_{ij} \times p_i$$

where:

$q_{ij}$ is the input $i$ in sector $j$ in ‘real’ term

$p_i$ is the basic price of input $i$
INTERMEDIATE FLOWS AT BASIC PRICES PLUS EXCISE TAX

Now we have

\[ X_{ij} = q_{ij} * p_i + s_{ij} \]

where:

\[ s_{ij} \] is the amount of excise tax on \( q_{ij} \)
The *ad valorem* Tax and Excise tax in the intermediate consumption flows

Now, the intermediate consumption flow is

\[ X_{ij} = q_{ij} \cdot p_i \cdot (1 + t_{ij}) + s_{ij} \]

or

\[ X_{ij} = (q_{ij} \cdot p_i + s_{ij}) \cdot (1 + t_{ij}) \]

where:

- \( t_{ij} \) is the *ad valorem* tax rate related to \( q_{ij} \cdot p_i \)
Peeling off the IO table

Reshaping the IO table thorough the removal of excise and ad valorem taxes
**Excise tax flows in IO matrix**

<table>
<thead>
<tr>
<th>$CI_1$</th>
<th>$CI_2$</th>
<th>$CI_3$</th>
<th>$FD$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{11}p_1 + s_{11}$</td>
<td>$q_{12}p_1 + s_{12}$</td>
<td>$q_{13}p_1 + s_{13}$</td>
<td>$c_1p_1 + s^1$</td>
<td>$q_1p_1 + s_{11} + s_{12} + s_{13} + s^1$</td>
</tr>
<tr>
<td>$q_{21}p_2 + s_{21}$</td>
<td>$q_{22}p_1 + s_{22}$</td>
<td>$q_{23}p_2 + s_{23}$</td>
<td>$c_2p_2 + s^2$</td>
<td>$q_2p_2 + s_{21} + s_{22} + s_{23} + s^2$</td>
</tr>
<tr>
<td>$q_{31}p_3 + s_{31}$</td>
<td>$q_{32}p_3 + s_{32}$</td>
<td>$q_{33}p_3 + s_{33}$</td>
<td>$c_3p_3 + s^3$</td>
<td>$q_3p_3 + s_{31} + s_{32} + s_{33} + s^3$</td>
</tr>
<tr>
<td>$VA_1$</td>
<td>$VA_2$</td>
<td>$VA_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_1p_1$</td>
<td>$q_2p_2$</td>
<td>$q_3p_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$II_1$</td>
<td>$II_2$</td>
<td>$II_3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$qq_1$</td>
<td>$qq_2$</td>
<td>$qq_3$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
\[ II_i \text{ excise tax row sum of sector } i \]

\[ qq_i \text{ column sum of sector } i \]

\[ II_i = s_{i1} + s_{i2} + s_{i3} + s^i \]

\[ qq_i = q_i p_i + II_i \]
IO table after the removal of the excise flows

<table>
<thead>
<tr>
<th></th>
<th>$CI_1$</th>
<th>$CI_2$</th>
<th>$CI_3$</th>
<th>$FD$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{11}p_1$</td>
<td>$q_{12}p_1$</td>
<td>$q_{13}p_1$</td>
<td>$c_1p_1$</td>
<td>$q_1p_1$</td>
<td></td>
</tr>
<tr>
<td>$q_{21}p_2$</td>
<td>$q_{22}p_{21}$</td>
<td>$q_{23}p_2$</td>
<td>$c_2p_2$</td>
<td>$q_2p_2$</td>
<td></td>
</tr>
<tr>
<td>$q_{31}p_3$</td>
<td>$q_{32}p_3$</td>
<td>$q_{33}p_3$</td>
<td>$c_3p_3$</td>
<td>$q_3p_3$</td>
<td></td>
</tr>
<tr>
<td>$VA_1$</td>
<td>$VA_2$</td>
<td>$VA_3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$IA_1$</td>
<td>$IA_2$</td>
<td>$IA_3$</td>
<td>$IA^c$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q_1p_1$</td>
<td>$q_2p_2$</td>
<td>$q_3p_3$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The removal of excise tax flows makes room for the excise tax row in the Value Added zone

Where

\[ IA_i = s_{1i} + s_{2i} + s_{13} \]

and

\[ IA^c = s^1 + s^2 + s^3 \]

the excise tax on final demand components not necessarily recorded in IO tables
### Ad valorem tax flows in IO matrix

<table>
<thead>
<tr>
<th></th>
<th>CI₁</th>
<th>CI₂</th>
<th>CI₃</th>
<th>FD</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>q₁₁p₁</td>
<td>q₁₂p₁</td>
<td>q₁₃p₁</td>
<td></td>
<td>c₁p₁(1 + t₁)</td>
<td>q₁p₁ + VATRS₁</td>
</tr>
<tr>
<td>q₂₁p₂</td>
<td>q₂₂p₂₁</td>
<td>q₂₃p₂(1 + t₂)</td>
<td></td>
<td>c₂p₂(1 + t₂)</td>
<td>q₂p₂ + VATRS₂</td>
</tr>
<tr>
<td>q₃₁p₃</td>
<td>q₃₂p₃(1 + t₃)</td>
<td>q₃₃p₃(1 + t₃)</td>
<td></td>
<td>c₃p₃(1 + t₃)</td>
<td>q₃p₃ + VATRS₃</td>
</tr>
<tr>
<td>VA₁</td>
<td>VA₂</td>
<td>VA₃</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>q₁p₁</td>
<td>q₂p₂</td>
<td>q₃p₃</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VATRS₁</td>
<td>VATRS₂</td>
<td>VATRS₃</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TOT₁</td>
<td>TOT₂</td>
<td>TOT₃</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Where VATRS stands for VAT Row Sum

\[ \text{VATRS}_1 = c_1 p_1 t_1 \]
\[ \text{VATRS}_2 = c_2 p_2 t_2 + q_{23} p_2 t_2 \]
\[ \text{VATRS}_3 = c_3 p_3 t_3 + q_{32} p_3 t_3 + q_{33} p_3 t_3 \]

\[ \text{VAT}_1 = 0 \]
\[ \text{VAT}_2 = q_{32} p_3 t_3 \]
\[ \text{VAT}_3 = q_{23} p_2 t_2 + q_{33} p_3 t_3 \]
\[ \text{VAT}^c = c_1 p_1 t_1 + c_2 p_2 t_2 + c_3 p_3 t_3 \]
### IO table when VAT flows are removed

<table>
<thead>
<tr>
<th>CI₁</th>
<th>CI₂</th>
<th>CI₃</th>
<th>FD</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{11}p_1$</td>
<td>$q_{12}p_1$</td>
<td>$q_{13}p_1$</td>
<td>$c_1p_1$</td>
<td>$q_1p_1$</td>
</tr>
<tr>
<td>$q_{21}p_2$</td>
<td>$q_{22}p_{21}$</td>
<td>$q_{23}p_2$</td>
<td>$c_2p_2$</td>
<td>$q_2p_2$</td>
</tr>
<tr>
<td>$q_{31}p_3$</td>
<td>$q_{32}p_3$</td>
<td>$q_{33}p_3$</td>
<td>$c_3p_3$</td>
<td>$q_3p_3$</td>
</tr>
<tr>
<td>VA₁</td>
<td>VA₂</td>
<td>VA₃</td>
<td>VAT</td>
<td>VAT</td>
</tr>
<tr>
<td>$q_1p_1$</td>
<td>$q_2p_2$</td>
<td>$q_3p_3$</td>
<td>$VAT_1$</td>
<td>$VAT_2$</td>
</tr>
</tbody>
</table>
Basic prices from the Leontief’s price equation

\[
\begin{bmatrix}
    p_1 \\
    p_2 \\
    p_3
\end{bmatrix}_{\text{basic}} =
\begin{bmatrix}
    1 - a_{11} & -a_{21} & -a_{31} \\
    -a_{12} & 1 - a_{22} & -a_{32} \\
    -a_{13} & -a_{23} & 1 - a_{33}
\end{bmatrix}^{-1}
\begin{bmatrix}
    v_1 \\
    v_2 \\
    v_3
\end{bmatrix}
\]
Price equation with excise tax (rates)

\[ a_{11} p_1 + a_{21} p_2 + a_{31} p_3 + a_{11} \alpha_1 + a_{21} \alpha_2 + a_{31} \alpha_3 + \nu_1 = p_1 \]

\[ a_{12} p_1 + a_{22} p_2 + a_{32} p_3 + a_{12} \alpha_1 + a_{22} \alpha_2 + a_{32} \alpha_3 + \nu_2 = p_2 \]

\[ a_{13} p_1 + a_{23} p_2 + a_{33} p_3 + a_{13} \alpha_1 + a_{23} \alpha_2 + a_{33} \alpha_3 + \nu_3 = p_3 \]
Price equation with excise tax (rates)

In the language of matrix algebra:

\[
\begin{bmatrix}
  p_1 \\
  p_2 \\
  p_3 \end{bmatrix}_{\text{excise}} = \begin{bmatrix}
  p_1 \\
  p_2 \\
  p_3 \end{bmatrix}_{\text{basic}} + \begin{bmatrix}
  1 - a_{11} & - a_{21} & - a_{31} \\
  - a_{12} & 1 - a_{22} & - a_{32} \\
  - a_{13} & - a_{23} & 1 - a_{33} \\
\end{bmatrix}^{-1} \begin{bmatrix}
  a_{11} & a_{21} & a_{31} \\
  a_{12} & a_{22} & a_{32} \\
  a_{13} & a_{23} & a_{33} \\
\end{bmatrix} \begin{bmatrix}
  \alpha_1 \\
  \alpha_2 \\
  \alpha_3 \\
\end{bmatrix}
\]

Here the excise tax produces a clear tax shifting
Price equation with ad valorem taxes (rates) such as European VAT

\[
\begin{align*}
    a_{11}p_1 + a_{21}p_2 + a_{31}p_3 + v_1 &= p_1 \\
    a_{12}p_1 + a_{22}p_2 + a_{32}p_3 (1 + t_3) + v_2 &= p_2 \\
    a_{13}p_1 + a_{23}p_2 (1 + t_2) + a_{33}p_3 (1 + t_3) + v_3 &= p_3
\end{align*}
\]
Firms may not be entitled to full deduction

- Firms exempted from VAT have no right to deduct the VAT paid on their taxed purchases
- Some intermediate transactions may be allowed only for partial deduction
- “Forfeit” (or “standardized”) systems
- The Agricultural special system
- Government Agencies
Price equation with ad valorem taxes (rates) such as European VAT

• In the language of matrix algebra:

\[
\begin{bmatrix}
  p_1 \\
  p_2 \\
  p_3
\end{bmatrix}_{vat} = \begin{bmatrix}
  1 - a_{11} & -a_{21} & -a_{31} \\
  -a_{12} & 1 - a_{22} & -a_{32}(1 + t_3) \\
  -a_{13} & -a_{23}(1 + t_2) & 1 - a_{33}(1 + t_3)
\end{bmatrix}^{-1} \begin{bmatrix}
  v_1 \\
  v_2 \\
  v_3
\end{bmatrix}
\]

• The ad valorem tax shifting is not additive to the basic price solution
Long run Price formation with \textit{ad valorem} and excise tax in sector j

\[ p_j = \sum_{i=1}^{n} a_{ij} p_i (1 + t_{ij}) + \sum_{i=1}^{n} a_{ij} \alpha_{ij} + v_j \]
Indirect tax flows in the Intermediate Consumption Table of the Italy IO Matrix

Some statistics

- Excise taxes are recorded in 73% flows
- Ad valorem taxes are recorded in 64% flows
Indirect Taxes between Intermediate Consumption and Final Demand

Intermediate Consumption is the tax base of:

- 14% of the *ad valorem* taxes
- 45% of the excise taxes
VAT in the Intermediate Consumption

• This ad valorem tax applied by the European Union Member States has been designed to be an indirect tax on Personal Consumption Expenditure.

• So called VAT ‘impurities’ extend VAT tax base to intermediate consumption flows

Main VAT bases in Intermediate Consumption

82% of the VAT tax yield comes from 5 sectors out of 59 (Eurostat standard Input-output EU country matrices)

- Financial intermediation services
- Insurance and pension funding services
- Services auxiliary to financial intermediation
- Public administration and defence services
- Health and social work services
Main VAT bases in Intermediate Consumption

Being mostly industry specific, the treatment of an *ad valorem* tax such as the European VAT deserves a special attention in modelling price equations.
After all

The construction of time series of excise and ad valorem tax matrices and the design of indirect tax scenarios is a challenging task up to the macroeconomic multisectoral model builder.
Thank you for your attention

Maurizio Grassini