1. An Interindustry Macroeconomic Model

The Inforum \textit{LIFT} (Long-term Interindustry Forecasting Tool) model is unique among large-scale models of the U.S. economy in that it is based on an input-output core, and builds up macroeconomic forecasts from the bottom up. In fact, this characteristic of \textit{LIFT} is one of the principles that has guided the development of Inforum models from the beginning. This is in part because we believe that the understanding of industry behavior is important in its own right, but also because this is how the economy actually works. Investments are made in individual firms in response to market conditions in the industries in which those firms produce and compete. Aggregate investment is simply the sum of these industry investment purchases. Decisions to hire and fire workers are made jointly with investment decisions with a view to the outlook for product demand in each industry. The net result of these hiring and firing decisions across all industries determines total employment, and hence the unemployment rate. In the real world economy pricing decisions are made at the detailed product level. Though we cannot work at this level, modeling price formation at the 2- or 3-digit commodity level certainly captures the price structure of the economy better than an aggregate price equation. In \textit{LIFT}, prices and incomes are forced into consistency through the fundamental input-output identity, and the aggregate price level is determined as current price GDP divided by constant price GDP.

Despite its industry basis, \textit{LIFT} is a full macroeconomic model, with more than 800 macroeconomic variables determined either by econometric equation, exogenously or by identity. The econometric equations tend to be those where behavior is more naturally modeled in the aggregate, such as the personal savings rate, or the 3-month Treasury bill rate. Hundreds of identities are used to collect detailed results into aggregates, and then to form other aggregate variables by equation or identity. For example, total corporate profits are simply the total of corporate profits by industry. An equation for the effective corporate tax rate is used to determine total profits taxes, which is a source of revenue in the Federal government account. Equations for contribution rates for social insurance programs and equations for transfer payments out of these programs can be used to study the future solvency of the trust funds. Certain macrovariables provide important levers for studying effects of government policy. Examples are the monetary base and the personal tax rate. Other macrovariables, such as potential GNP and the associated GNP gap provide a framework for perceiving tightness or slack in the economy.

Since its inception in the early 1980s, \textit{LIFT} has continued to develop and change. We have learned more about the properties of the model through working with clients, and in doing our own simulation tests. We have learned about the behavior of the general Inforum type of model, from work with our partners in other countries. Finally, through many experiments, we have learned that many principles of economics, while attractive theoretically, are difficult to implement practically. We will continue to experiment, and share ideas, and bring the model closer to our vision of what it should be.
2. The *LIFT* Database

As described in the first section, *LIFT* is an interindustry model in that it most equations are estimated at an industry level, and the price and output solution by industry uses the fundamental input-output identities. However, the macroeconomic properties of the model are also important, and we continue to question and test the behavior of the model in response to exogenous shocks.

The database of *LIFT* therefore consists not only of input-output matrices, and vectors of expenditures, value added and employment, but also numerous macroeconomic variables. Listed below are some characteristics of the current model database.

- The input-output tables are based on the 1987 U.S. benchmark input-output study. All constant price data is in 1987 constant dollars. Data from the 1992 benchmark study has been incorporated in the investment bridge, consumption bridge, government bridges, and in various data vectors. The 1987 data has been updated through 1997 based on information about output and final demands.

- The model has 97 input-output commodity sectors, 92 personal consumption expenditure categories, and 55 equipment investment categories. The value added sectoring is comprised of 51 industries.

- The personal consumption equations use the perhaps adequate demand system (PADS) form. This form specifies the consumption for 92 categories of goods and services as a joint system. The 92 categories are divided into functional groups and subgroups both to make the estimation more tractable, and because it is a sensible approach.

- Defense spending is linked from 25 categories in the National Income and Product Accounts (NIPA) to the 97-sector level by a detailed defense bridge. Nondefense federal spending uses a bridge with 8 columns that relate to NIPA categories and state and local education, health and other each have a bridge of 7 columns each. Government construction is handled clearly and separately from private construction, or from other government spending. All sectors of government have adopted the distinction made in the NIPA between government consumption and government investment. There are separate equations for government capital consumption, based on estimated government capital stocks.

- The wage equations are based on expected inflation and industry labor productivity growth.

- Nominal GDP is same whether calculated as the sum of income or expenditure categories. This allows the four major balance identities to hold.

- All macrovariables have been updated to somewhere between 1999 and 2000. All NIPA data is updated through 2000.

- There are a total of almost 200 vectors and matrices whose role in the final model solution can be viewed in *G7*, or printed using the *Compare* program. All expenditure vectors are also calculated and stored in current prices, to make verification of the GDP identity easier.
3. A Turn Through the Model

Figure 1 shows the main program of the model in outline. The name of each function is shown, with comments to the right. The outside loop indicates the range of years for which the model will solve. For each year, we first make first guesses at some important endogenous variables, such as output and prices by industry, import shares, and many macrovariables. Fixes for exogenous variables are also done here. Then begins the model loop. This loop will continue to run, until outputs and other variables converge.

The model loop begins on the real side, where the expenditure components of GDP are calculated in 1987 constant dollars. Before starting the expenditure calculations, estimates of final demand prices are made, based on the best current estimate of producer prices by product. Next, the savings function is called, to determine how much of real disposable income will result in total expenditures on consumption. From total expenditures, total population and an income distribution function, we calculate the distribution of per-capita expenditures for five income classes. The cross-section equations of consumption per age-weighted population are calculated next. Once this is done, relative consumption prices, age-weighted population and consumption per age-weighted population are combined in the PADS function to get consumption by category. PADS allows the classification of consumption goods into related expenditure groups. For example, the first 14 consumption categories are in the food group. The first 3 of these are in the meat and poultry subgroup. PADS also allows for group, sub-group and individual commodity price parameters. Motor vehicles prices affect the demand for public transportation, since motor vehicles and public transport are substitutes.

After personal consumption, exports are calculated. If the model is run with the Inforum bilateral trade model (BTM), then exports are exogenous. However, if one wants to relax the dependence on BTM, then export equations are available which use information from BTM in the form of weighted foreign demands and foreign prices. The equipment investment equations are based on a Diewert cost function, that models the substitution (or complementarity) of equipment capital with labor and energy. The equations use a cost of capital measure that includes real interest rates, present value of depreciation, investment tax credit and corporate profits tax. The construction equations are for the roughly 20 categories of private construction. Though each has a different form, common variables are interest rates, disposable income and sectoral output.

Federal and state and local consumption and investment expenditures are specified exogenously in real terms, but LIFT allows for detailed control of these expenditures. For example, defense purchases of aircraft can be specified independently of missiles, ships or tanks. Capital consumption allowances of government are endogenous, based on depreciation of government capital stock, which is also calculated in the model.

At this point, all final demand expenditure categories except for imports and inventory change have been calculated. This means we are ready to use the Seidel input-output solution to solve jointly for output, imports and inventory change. Note that the A-matrix coefficients are specified to change over time, according to trends for each row. However, individual coefficients can be fixed, to model changes in price or technology.

The arrow in the diagram indicates that this is the end of the investment output loop. This loop is helpful in obtaining consistency between construction and equipment investment and output. Both of these categories of demand depend on output, but since they generate final
demand, they also contribute to output. It’s best to bring investment and output into agreement before moving on to calculate employment and prices.

Once the investment output loop has converged, the labor productivity equations can be calculated, which forecast the ratio of output to hours worked. Then the average hours equations are solved, which determine the average hours per employed person per year. Together, the productivity, average hours and output forecast generate employment by industry in the private sector. Adding in exogenous projections of government and domestic employment, we obtain total civilian employment, on the establishment basis, which is then adjusted to the household basis. Subtracting this employment from exogenous labor force projections yields unemployment, and then we calculate the unemployment rate. This is a pivotal variable in the model. Now the real side of the model is finished. For almost all of the equations in the real side, we needed information on relative prices (and the aggregate price level as well, to generate real disposable income). However, until the price side of the model has been run, these prices must only be guesses. So next the model turns to the important job of forecasting prices.

Prices are forecasted as a markup over unit intermediate and labor costs. However, all components of value added are calculated first. Some are then scaled so that value added by commodity and prices are consistent. The first item of business is to get hourly labor compensation by industry, which we will call the “wage rate”, although it also includes supplements. The “wage” equations relate the growth of the wage rate to growth in the ratio of M2 to GNP, expected inflation, and the growth in labor productivity. Multiplying the wage rate by the total hours worked per industry gives total labor compensation per industry. Next the total labor compensation is split into wages per se, and supplements such as employer contributions for social insurance and other labor income.

Labor compensation is the largest component of income, usually about 60% of GDP, and certainly has the largest effect on prices. However, it is also important to determine the components of capital income. Corporate profits are needed to be able to calculate corporate profits taxes, and retained earnings and capital consumption allowances are the large components of business savings, which is an important part of the savings-investment identity. Furthermore, dividends, proprietors’ income, interest income and rental income all contribute to personal income.

The corporate profits equations relate the ratio of economic profits over labor compensation to a measure of aggregate tightness (the GNP gap), changes in industry output, and the prices of oil and agriculture as supply variables. Economic profits are defined as profits plus the inventory valuation adjustment plus the capital consumption adjustment. The proprietors’ income equations take many forms, but typical right hand side variables are measures of tightness, the change in industry output, the change in GNP, and the change in the aggregate deflator. The two other large components of value added that have industry equations are corporate and non-corporate capital consumption allowances. The main explanatory variables are book value estimates of capital stock, which are formed by cumulating current price investment.

The equations for net interest, rental income, business transfer payments, inventory valuation adjustments, and government subsides are aggregate equations, which are then shared out to industries, based on the share in the last year of data. Indirect business taxes are determined by multiplying exogenous indirect tax rates by output by industry.
Before calculating prices, value added by industry is summed to total value added, and then passed through the product-industry bridge, to obtain value added by product. This product-industry bridge is derived from the make matrix, which shows the distribution of the production of any given product across various industries. We assume that value added can be allocated by the same distribution, and so use this bridge to translate value added by industry into value added by commodity, and vice versa. Once value added at the product level has been obtained, commodity prices are calculated using an input-output equation for prices, that also takes account of the import composition of intermediate consumption.

4. Model Description

It would take a book to describe the components of LIFT in a way that does justice to the work that more than a dozen people have contributed to the model. However, I shall attempt in this section to give more than a cursory view of some of the important sectoral equations in the model. The accountant and some of the more important macroeconomic equations are also presented.

4.1 The PADS Personal Consumption Equations

Personal consumption is the largest single component of GDP on the product side, and therefore it is important to pay special attention to the properties of the consumption functions. In LIFT, aggregate consumption is determined by the savings rate, in violation of the Inforum principle of building up macroeconomic variables from the detail. Theoretically, savings is just another consumption good, representing future consumption, and could logically be modeled in combination with the other goods. Alternatively, one could model only consumption of goods and services, and determine savings as a residual. However, in a long-term forecast, total consumption could move far away from disposable income, leading to strange values for the implied savings rate. We have found it more convenient to keep a separate equation for the savings rate, and then scale total personal consumption to the total expenditures implied by the savings rate.

The perhaps adequate demand system (PADS) borrows something of its name from the almost ideal demand system (AIDS), a well-known and often used system of consumption. However, PADS doesn’t try to mimic AIDS’ properties. A known failing of AIDS is that increasing real income must ultimately drive the consumption of some goods negative, unless it has no effect at all on the budget shares! PADS doesn’t have this property, being derived from an earlier form introduced by Almon (1979), with a multiplicative relation between the income terms and price terms:

$$\chi_t = \left( a_i + b_i \frac{y_t}{p_t} \right) \prod_{k=1}^n P_k^{C_i}$$

The left hand side of this equation is per capita consumption of product $i$ in period $t$ and $a_i$ is a function of time. The $y$ in this equation is nominal income per capita; $p_k$ is the price index of product $k$ and $P$ is the overall consumer price index, defined by:

$$P = \prod_{k=1}^n P_k^{y_t}$$
where $s_k$ is the budget share of product $k$ in the base period of prices. The $c_{ik}$ are constants to be estimated, and are related to the own and cross-price elasticities. The PADS form is derived from this simpler form, but the modeler allocates consumption goods into groups and subgroups, based on their role in consumption. This effectively reduces the number of parameters to be estimated, and provides a framework for organizing such a large number of consumption goods.

PADS can be estimated directly with time series data, in which case $Y$ is simply total expenditures. Alternatively, a two-stage approach can be taken, where cross-sectional equations are used to estimate an Engel curve, adult-equivalency weights, and effects of various demographic characteristics for each good, and then the left hand side prediction from this equation can be used as the “income” term in the time series equation. This is the approach we follow in LIFT.

The cross-section equations are of the following form:

$$C_i^* = \left( a + \sum_{k=1}^{K} b_k Y_k + \sum_{l=1}^{L} d_l D_l \right) \left( \sum_{g=1}^{G} W_g n_g \right)$$

where:

$C_i^*$ = household consumption expenditures on good $i$

$Y_k$ = the amount of per-capita income (expenditures) within income category $k$

$D_l$ = dummy variable used to show membership in the $j$th demographic group

$n_g$ = number of household members in age category $g$

$W_g$ = adult equivalency weights

$K$ = the number of income groups

$L$ = the number of demographic categories

$G$ = the number of age groups

The demographic categories $D$ include region of residence, family size, working status of spouse, college education, and age of household head, all estimated using dummy variables. The left-hand side result of this equation is known to us by the name of “C-star”. The two terms in the first factor of the equation are the “piecewise linear Engel curve” and the demographic term. The second factor of this equation is the age-weighted population.

The PADS equations take the form:

$$x_i(t) = \left[ a_i(t) + b_i \left( \frac{C_i^*}{P} \right) + c_i \left( \frac{C_i^*}{P} \right)^{\lambda_i} \prod_{i=1}^{l} \left( \frac{p_i}{p_k} \right)^{\lambda_{is} \mu_s} \left( \frac{p_i}{p_G} \right)^{\mu_i} \right]^{\nu_i}$$

where:

$C_i^*$ = cross-section expenditures for corresponding cross-section category $c$
\[ P = \text{overall consumption price index} \]
\[ p_i = \text{the price of good } i \]
\[ p_G = \text{the average price index of group } G \]
\[ p_g = \text{the average price of subgroup } g \]
\[ \lambda_i = \text{individual good price response parameter} \]
\[ \mu_G = \text{group price response parameter} \]
\[ \nu_g = \text{subgroup price response parameter} \]

The PADS equations have been estimated for 92 commodities for the U.S. Estimation and results are described in Almon (1997). After consumption by category has been solved for in the model, this vector is passed through a consumption bridge, to obtain consumption by input-output commodity. This bridge also serves the function of stripping off trade and transportation margins to generate demand for the trade and transportation industries.

4.2 The Bilateral Trade Model, or the Export Equations?

The Inforum bilateral trade model (BTM) forecasts bilateral trade flows for 120 commodities, by 14 trading partners and two regions covering the rest of the world. The forecasting equations are based on annual OECD and UN data on international trade by commodity and country of origin. For 13 of the trading partners, Inforum models are available, which forecast imports and domestic prices endogenously. Given each country’s imports of a given commodity, BTM decides from whom that commodity will be imported, based on relative prices between countries, and relative rates growth of capital stock for the commodity between countries. After BTM has solved, it provides forecasts of exports and average foreign prices back to each national model, which are then treated as exogenous assumptions for that model. Since each model has its own sectoring plan, different from the 120 commodity sectoring, the model forecasts of imports, capital stock and prices must be converted to the 120 commodity sectoring. Then the BTM forecasts of exports and foreign prices for each country must be converted back to that country’s native sectoring.

BTM provides a rich level of detail for the study of international trade, and the modeler can study impacts of changing exchange rates, changing prices, or even directly changing import shares exogenously. However, for the person running an individual country model in isolation, the exogenous export projection from BTM is sometimes too constraining. Part of the macroeconomic adjustment one expects from a tight economy is a rise in interest rates, which raises the value of the dollar. There is also a slight increase in domestic prices that makes exports less competitive. Without running the entire BTM several iterations, exports are effectively fixed, removing this stabilizing effect.

For this reason, we have also estimated export equations such as described in Nyhus (1991), used in the previous Inforum international system. The form of these export equations is:
\[
E_i = \left( b_0 + b_i \sum_{k=1}^{n} W_k m_{it} \frac{p^d_{it}}{m_{it0}} \right) \left( \frac{p^d_{it}}{p^f_{it}} \right)^q \\
\]

where:

- \( E_i \): U.S. exports of commodity \( i \) for year \( t \)
- \( W_k \): the fraction of U.S. imports which went to trading partner \( k \) in the base year (1987)
- \( m_{it} \): imports to country \( k \) for commodity \( i \) in year \( t \)
- \( m_{it0} \): imports to country \( k \) for commodity \( i \) for the base year
- \( p_s \): a moving average of the domestic price
- \( f_s \): a moving average of a weighted exchange rate adjusted price of competing exporters.

In contrast to the older \( LIFT \) equations of this form, these equations were all estimated with data from the BTM database, and the forecasts from BTM are still used to obtain the foreign import demands and foreign prices. However, this form allows the \( LIFT \) equations to adjust independently from BTM. These equations, like all others in the model, can be turned on or off.

### 4.3 The Equipment Investment Equations

Equipment investment is also an important component of GDP, playing a major role in the medium-term cyclical behavior of the economy, as well as contributing to capacity for further long-term growth. \( LIFT \) forecasts purchases of equipment investment for 55 industries comprising the U.S. economy. Sales of investment goods at the 97 commodity level are then determined by passing equipment investment by buyer through the investment bridge matrix. Thus the model is capable of determining not only the direct and indirect impacts of a given increase in demand for some good, but also the investment purchases stimulated by that demand, and the capital goods inputs need to produce those investments.

The investment equations are estimated in a two-stage, three equation framework. Factor demands for equipment capital, labor and energy are estimated simultaneously. In the first stage, optimal capital-output, labor-output and energy-output ratios are estimated. In the second stage, the parameters from the first stage are treated as fixed, and equations for net investment, labor and energy are estimated. In this stage, investment is based upon a distributed lag on past changes in output, whereas labor and energy demand are based upon a distributed lag of levels of output.

The first stage equation for the optimal capital-output ratio is obtained by using Shephard’s Lemma with a generalized Leontief cost function with equipment, labor and energy to obtain:

\[
\left( \frac{K}{Q} \right)_t^* = e^{-\alpha_e t} \alpha_e t \sum_{j=0}^{K} b_{jk} \left( \frac{p_j}{p_K} \right)^{1/2} 
\]
where:

- $K = \text{capital stock}$
- $Q = \text{output}$
- $\left( \frac{K}{Q} \right)^* = \text{the optimal capital-output ratio}$
- $p_j = \text{price of factor } j, \text{ where } j = K, L, E$
- $t_1 = \text{time trend}$
- $t_2 = 2^{\text{nd}} \text{ time trend, starting in } 1970$

This equation is used in a three equation system to fit the historical capital-output, labor-output and energy-output ratios.

The equation for net investment is derived from the first difference of the optimal capital stock equation and can be expressed by:

$$N_t = e^{-\alpha_1 t - \alpha_2 t^2} \left[ \sum_{j=K,L,E} b_{kj} \left( \frac{p_j}{p_K} \right)^{1/3} \right] \sum_{j=0}^3 w_j \Delta Q_{t-j}$$

where:

- $N_t = \text{net investment}$
- $\Delta Q = \text{the change in output}$

The price of capital $p_k$ is the commonly used neoclassical measure:

$$p_k = \frac{p_{eq} (r + dep)(1 - tz - c)}{1 - T}$$

where:

- $p_{eq} = \text{the equipment price deflator for this purchasing industry}$
- $r = \text{the real AAA bond rate}$
- $dep = \text{the average depreciation rate for this industry}$
- $T = \text{the effective corporate tax rate}$
- $z = \text{the present value of depreciation of one dollar worth of investment}$
- $c = \text{the investment tax credit}$
Replacement investment is determined by multiplying the optimal capital output ratio by the losses to capacity (as the level of optimal output given the current capital stock) occurring in the current year. Since the optimal capital-output ratio is a function of relative prices, price change affects both the demand for net investment and replacement investment.

4.4 Private Construction by Type

The equations for private purchases of plant and other structures are for 19 categories of construction available from the NIPA. These purchases are generally aggregated into two major divisions. Residential construction consists of single- and multi-family homes, and additions and alterations. Non-residential construction is comprised of a motley of different types: hotels, industrial buildings, office buildings, schools, farm buildings, oil wells, railroads, telephone and communications, electric and gas utilities, and petroleum pipelines. The residential equations are estimated in per-capita form, and based on disposable income per capita, the mortgage interest rate, and the percent of households of home-buying age. The non-residential constructions are each unique, but often based on the output of the related industry or group of industries, the relative price of the related industry (especially in the case of oil and drilling rigs), interest rates, and a variety of demographic variables. Some of the equations also use a measure of capital stock of structures of that type, to model replacement investment needs.

4.5 Government Consumption and Investment

In 1996, the NIPA changed their treatment of government expenditures. What used to be classified as purchases are now divided into consumption and investment categories, based on the average life of the good, as well as corresponding treatment of the good in private industry. For example, investment purchases of aircraft for defense are the new aircraft, as well as replacement components such as large engines or upgraded guidance systems. Consumption purchases include smaller replacement parts, tires and jet fuel.

Only consumption purchases are included in the presentation of the government revenues and expenditures. However, investment is accumulated into a book value stock, and the depreciation of this stock is the capital consumption of government. This capital consumption is part of current consumption expenditures.

LIFT has adopted this new accounting scheme, and we have developed an accounting for the government capital stock, and estimated capital consumption equations that relate capital consumption to the calculated depreciation from this stock. This capital consumption is part of the current government budget, and also shows up in the non-corporate capital consumption vector in the income side of the model.

Aside from capital consumption, the other categories of government consumption and investment are exogenous. However, these variables can be fixed at a fairly detailed level. For example, state and local purchases of structures can be fixed for 11 categories of construction, for education, health and other, for a total of 33 categories. If more aggregate control is desired, the total value of construction for each category of government can be fixed, and it will be allocated to construction by type by means of a bridge matrix.
There are five categories of government spending: federal defense, federal nondefense, state & local education, state & local health and state & local other. For each category, there is a bridge that translates purchases by type to purchases by input-output commodity. For example, the bridge for federal defense spending has 97 rows and 25 columns.

4.6 Output, Imports and Inventory Change

Central to the solution of the real side of an Inforum model is the calculation of output by industry. The sources of demand by commodity arising from personal consumption, equipment investment, structures investment, government and exports are first added together to form a vector of final demand. Then the Seidel iterative technique is used to jointly solve for domestic output, imports and inventory change by industry.

The Seidel algorithm in LIFT solves by iteratively calculating

\[
q^{k+1}_i = \frac{\sum j a_{ij} q^{k+1}_j + \sum j a_{ij} q^k_j + v_i - m_i + d_i}{1 - a_{ii}}
\]

where:
- \(q_i^k\) = the solution for output of commodity \(i\) in iteration \(k\)
- \(a_{ij}\) = the direct input-output coefficient from commodity \(i\) to commodity \(j\)
- \(f_i\) = the sum of final demand for commodity \(i\) excluding imports, inventory change, and discrepancy
- \(v_i\) = inventory change for commodity \(i\)
- \(m_i\) = imports for commodity \(i\)
- \(d_i\) = final demand discrepancy for commodity \(i\)

A triangulation ordering typically optimizes the loop over commodities, so that the solution is as recursive as possible. Within the loop for each commodity, the equations for inventory change and imports are also calculated. One may think of domestic output, inventory change and imports as being three alternative sources of supply for total requirements. When calculating them together in the Seidel loop, a consistent and reasonable solution is usually reached fairly quickly.

One complication is the final demand discrepancy, which is the approach we have taken to handle the inconsistencies between input-output tables, final demand data and output data. The discrepancy \(d\) is formed in the last year of output data as \(d = q - Aq - f - v + m\), where \(d\) is the final demand discrepancy, \(q\) is output, \(A\) is the direct requirements matrix, \(f\) is other final demand, and \(v\) is inventory change and \(m\) is imports. In the forecast periods, the discrepancy is kept constant, and added back in during the Seidel solution.

The import equations are based on domestic demand, calculated in the regressions as \(dd = q + m - x\), where \(dd\) is domestic demand, and \(x\) is exports. However, when solving for imports, \(m\)
and \( q \) are still unknown, so we use instead \( dd = Aq + f + v + d \), where \( q \) is the best current guess of output for the current iteration.

The form of the inventory equations is:

\[
v_i = \beta_0 + \beta_1USE_i + \beta_2\Delta USE_i
\]

where:

\[
USE_i = q_i(1 - a_\phi) + m_i - v_i
\]

### 4.7 Labor Productivity, Average Hours Worked and Employment

The growth of labor productivity is probably the single most important determinant of the growth of real per-capita income in the economy. The labor productivity equations we use can be written as:

\[
\ln \left( \frac{q}{h} \right) = \beta_0 + \beta_1t_1 + \beta_2t_2 + \beta_3q_{up} + \beta_4q_{down}
\]

where:

- \( q = \) output
- \( h = \) hours worked
- \( t_1 = \) a simple linear time trend
- \( t_2 = \) a second time trend, starting in 1972
- \( q_{up} = dq \), when positive, 0 otherwise
- \( q_{down} = -dq \), when \( dq \) negative, 0 otherwise
- \( dq = \ln q_i - \ln q_{peak_{t-1}} \)
- \( q_{peak_i} = q_i \), if \( q_i > q_{peak_{t-1}}(1 - spill) \), otherwise \( q_{peak_{t-1}}(1 - spill) \)
- \( spill = \) depreciation rate of capacity, both in the sense of capital and “hoarded” labor

The second time trend picks up a change in the rate of labor productivity growth that began sometime in the 1969 to 1973 period. The \( q_{up} \) and \( q_{down} \) terms model the increase in labor productivity that is observed in periods of increasing output, and vice-versa. This phenomenon of procyclical labor productivity is sometimes associated with “labor hoarding”, where firms retain trained workers in periods of slack output. When output increases again, they put the hoarded labor back to work before making new hires. The \( q_{peak} \) variable attempts to measure capacity output, both in the sense of capital and “hoarded” labor. Work is currently underway to estimate the effects of vintages of investment on labor productivity.

The equations for hours worked relate annual hours worked per employee to a time trend and cyclical changes in output, much like the labor productivity equations. Therefore, they are also essentially time trends.
Hours worked by industry can be obtained by dividing output by productivity. Then employment by industry is simply total hours divided by average hours worked per employee. This yields hours and employment for all industries comprising the private economy. Public sector employment, domestic employment and rest of world employment are specified exogenously.

4.8 Measures of Tightness

Before turning to some of the equations in the price-income side of the model, it would be helpful to discuss the alternative measures of tightness and slack in the economy. These are: the unemployment rate, or the difference of the unemployment rate from some specified “natural” rate; the GNP gap, which is an index that rises above 100 when the economy is tight; and capacity utilization, which is currently available from the Federal Reserve for only the mining, manufacturing and utilities sectors, and is a measure of how intensively the capital stock of various industries is being used. Although one may argue that the level of “core” inflation is determined generally by average money supply growth, the acceleration or deceleration of inflation around this core rate is surely determined by periods of relative tightness or slack.

The unemployment rate has long been considered a useful variable for indicating the pressure of aggregate demand relative to aggregate supply. Some variant of a Phillips curve has been included in the price equations of just about every macromodel since the early aggregate Keynesian models. As a short-term indicator, the unemployment rate is extremely useful. Over a longer period however, the natural rate may drift, due to demographics and other factors. Therefore, when we use this variable, it is usually in reference to the natural rate, for which there are published historical time series. In the model forecast, the natural rate is exogenous, usually set to about 5.5%. The unemployment rate has played an important role in the LIFT savings rate equation, but it is not in the current version.

The GNP gap is defined as \( \frac{\text{gnp}}{\text{gnp}} \times 100 \), where \( \text{gnp} \) is potential GNP. The concept of potential GNP is simple: it is that level of GNP at which the economy is neither running above or below its capacity, as determined by labor force growth, labor participation, and labor productivity. Specifically, we estimate the regression, where \( \beta_1 \) is constrained to 1:

\[
\ln \text{gnp} = \beta_0 + \beta_1 (\ln \text{smprd} + \ln \text{smlfc} + \ln \text{smhrs})
\]

where:

\( \ln \text{gnp} \) = the log of actual gnp  
(\ln \text{smprd} \) = the log of a five-period moving average of aggregate labor productivity  
(\ln \text{smlfc} \) = the log of smoothed labor force, estimated as a five-year moving average of the labor force participation rate times the working-age population  
(\ln \text{smhrs} \) = the log of a five year moving average of aggregate hours

The antilog of the predicted value of this regression is used as the estimate of potential GNP. We have found the GNP gap to be a useful alternative to the unemployment rate in the price
equations, the profit equations, and several others. The current version, which uses moving averages over five previous years, is backward-looking, and ignores capital stock. However, it is quite stable for long-term forecasting.

A third measure of tightness, for which data is available at the industry level, is the Federal Reserve measure of capacity utilization. The aggregate level of capacity utilization is a remarkably good indicator of the short-term cyclical prospects for inflation, performing better than the unemployment rate or the GNP gap. However, the definition and modeling of capacity is difficult, although we have made some experiments in this area.

4.9 The Wage Equations

In the price-income side of the model, the wage equations are really the backbone, for labor compensation comprises the largest share of income, and the most significant contributor to the core inflation rate is wage inflation. It is perhaps appropriate that it is here that we introduce the growth of money into the model, as the long-run determinant of average inflation.

The wage equations in LIFT are estimated in a stacked system, and the left-hand side variable is the percent change in the hourly labor compensation in each industry. The wage equations are estimated as:

$$dw_i = \beta_1 p_j + \beta_2 dlp_i$$

where:

$$dw_i = \ln(wag_i) - \ln(wag_{i-5})$$, and $$wag_i$$ is the labor compensation rate for industry i

$$p_j$$ is a five-year weighted average of the percent change in the growth of M2/real GNP

$$dlp_i = \ln(q_{i}/h_{i}) - \ln(q_{i-1}/h_{i-1})$$, the percent change in industry labor productivity

Although the main motive of introducing the monetary aggregate into this equation is to provide a mechanism whereby money affects prices, there is also a rationale supported by anecdotal evidence. This evidence suggests that when the money supply increases more rapidly, it stimulates aggregate demand. This creates pressure in the labor markets, which puts upward pressure on wages. An alternate story is the rational expectations version, that workers bid up wages in expectation of the higher inflation which they know will be generated by the money supply growth. From earlier experiments, we found that putting money supply growth in the demand equations, and the unemployment rate in the wage equations led to unsatisfactory results. The method we now use allows for a more direct and reliable influence of money on prices.
4.10 The Price Equations

The design of the price and income block in Inforum models has been dominated by two opposing approaches. The first has been to forecast prices directly, and then determine value added by identity. The price equation approach is quite common in other industry models. The alternate approach is to forecast value added per unit of output, or value added directly, and then calculate prices through the second fundamental input-output identity:

\[ p' = p'A + v \]

4.11 The Product-Industry Bridge

The input-output table and the vectors of final demand expenditures are compiled for the model at the 97 commodity level. However, the categories of value added are compiled for 51 industries. When value added is used to calculate prices using the equation \( p' = p'A + v \), it must first be converted to the commodity level, which is basis of the A matrix and the price vector. If the price equations are used, or if prices are fixed exogenously, then industry value added must be modified so as to be consistent with these commodity prices. For these purposes and others, we make use of the product to industry bridge.

The product-industry bridge is derived from the make matrix, which in the case of LIFT has 51 rows (industries) and 97 columns (commodities). The bridge is stored and updated in flows, which enables converting in both directions.

In the base year, which we designate as year 0, the following relationship holds:

\[ \sum_{industry} g_i^0 \frac{V_j^0}{\sum_i V_j^0} = vaa_j^0, \text{ for all } j \text{ commodities} \]

where:

- \( g_i^0 \) = the total vector sum of 13 categories of value added by industry, including a discrepancy column, so calculated to that the above relationship holds
- \( V_j^0 \) = the product-industry bridge matrix for the base year
- \( vaa_j^0 \) = the commodity level vector of value added allocated. In the base year, this is the vector of value added in the input-output table, which satisfies the fundamental identities

An important concept to grasp is that of real value added weighted output, or \( revawo \). Denote the ratio \( V_j^0 / q_j^0 \) the value-added fraction. This is the share of total output of commodity \( j \) in the base year, accounted for by output produced in industry \( i \). To transfer output through the bridge, we multiply each output by its value added fraction, and distribute the result to the various industries in their proportion to that commodity in the base year. The result is \( revawo \), and may be written:

\[ revawo_i' = \sum_j \frac{V_j^0}{q_j} \cdot q_j' \]
In the years beyond the base year, the $V$ matrix is updated as follows:

$$V'_i = V'_0 q'_j g'_i / q'_j revawo_i$$

In this equation $g'_i$ is the sum of industry value added plus the value added discrepancy column. This equation adjusts the bridge matrix so as to prorate each industry’s value added back to products in accordance with that product’s contributions to the industry $revawo$. To obtain value added by product we then just sum down the column of the updated bridge.

If prices are forecast directly, as we are currently doing, we first calculate commodity value added from prices and intermediate cost and price discrepancy:

$$vaa = p \cdot q - \sum p_w a_i q - pdisc$$

where:

$p_w$ = the weighted foreign and domestic price

$pdisc_j$ = the price discrepancy, calculated as $p' - p'A - v$ in the last year of price data

Next the $vaa$ vector is passed through the bridge in the reverse direction to obtain value added by industry. Finally, several components of value added are scaled so that total value added by industry is equal to the desired total. These components are corporate profits, proprietor’s income, and corporate and non-corporate consumption allowances.

### 4.12 The Accountant

Even if forecasting the prices directly we still need to develop consistent estimates of the other components of income besides labor compensation. Rental income, interest income, proprietor’s income and that part of profits paid out as dividends all contribute to personal income. Corporate profits taxes and indirect business taxes contribute to the revenue of governments. Capital consumption allowances and retained earnings are part of business savings, which is an important component of national savings. It is the job of the Accountant to aggregate the components of value added on the price income side and obtain the aggregate variables needed to state the relationships between GNP, national income, personal income and disposable income. Along the way, the important components of the household and government balance sheets are obtained. The Accountant also forms aggregates of the expenditure vectors on the real side of the model, and forms implicit deflators from current and constant price aggregates.

The operation of the Accountant can be viewed in several stages. In the first stage, several aggregates of income are created from the price-income side, and summed to form nominal GNP. Factor imports are added and factor exports subtracted to obtain GDP. Supplements to labor compensation such as employer contributions are first aggregated across industries and then distributed to different funds based on exogenous ratios. Components of other labor income are also calculated in this stage. The second stage computes capital consumption adjustments,
and forms proprietor’s income, rental income and profits with and without capital consumption adjustments and inventory valuation adjustments. In the third stage, national income is formed by summing labor compensation, proprietor’s income, rental income, corporate profits and interest income. Corporate profits, net interest and contributions for social insurance are subtracted, and transfer payments, personal interest income, personal dividend income, and business transfer payments are added back in to obtain personal income. This stage is quite lengthy due to a detailed set of identities and regression equations calculating the different components of transfer payments and interest payments. In the fourth stage, the components of federal and state and local receipts and expenditures are calculated. In the fifth and final stage, personal taxes and non-tax payments are removed to obtain disposable income. At this point, the loop in the model has been closed, and it returns to the real side, with the new guess at disposable income.

After prices have been computed, value added by commodity is recalculated as current price output less current price intermediate cost, and a discrepancy term. The product-industry bridge is used to recalculate value added by industry. Corporate profits, proprietors’ income and capital consumption allowances are then scaled so that total value added by industry is correct. All that remains to be done in the price side is to calculate some other prices based on the domestic output prices. The price income loop is usually iterated at least twice, to make sure all parts are consistent with each other.

Once the price income loop has finished, it is now the job of the Accountant to summarize the industry results of the model, determine the household and government receipts and expenditures, and forecast important financial variables. For example, the Accountant forms personal income as the sum of wages and salaries, other labor income, proprietors’ income, rental income, dividend income, interest income and transfer payments, less social insurance taxes. Then it subtracts personal tax and non-tax payments to arrive at personal disposable income. The model is now ready to return to the beginning of the real side, and solve again with this much better guess at disposable income.

The model iterates over the real side, price income side and the accountant several times until it reaches convergence. After this, several capital stocks are calculated to prepare for the next year.

4.13 The Personal Savings Rate

The personal savings rate, like the unemployment rate, is a pivotal variable in the model, especially since it determines personal consumption, the largest component of GDP on the expenditure side. However, understanding the behavior of the personal savings rate in the U.S. is a thorny task, and we have been pricked by it. This equation has gone through many revisions since the first version of the model. Common variables used were the unemployment rate, usually with a coefficient of close to –1, and the share of motor vehicles in personal consumption, also with a coefficient of –1. The argument for using motor vehicles share was that consumers may view purchases of consumer durables generally as a substitute for savings. The reason for using the unemployment rate is to introduce an automatic stabilizer in the model.

The current equation is rather simple,

\[ \text{savrat} = \beta_0 + \beta_1 (\text{gapgap} - 100) + \beta_2 rtb + \beta_3 \text{dummy} \]
where:

\[ rtb = \text{the treasury bill rate} \]
\[ dummy = \text{a dummy variable starting in 1986} \]

The parameter \( \beta_1 \) is constrained to be about .75. The dummy variable takes the value 1 after 1986, when a significant decrease in the U.S. personal savings rate was observed. Despite its importance in the structure of the model, the savings rate equation is in trouble. The U.S. has recently seen a personal savings rate become negative. The economy is very strong now, so this would usually be a cause for the savings rate to be higher. However, there has always been something problem of direction of causality in our specification of the savings equation. In the current U.S. situation, something is making consumer spending very strong, and this is both stimulative to the economy, as well as resulting in negative measured savings. It could be the wealth effect from the strong stock market, or the price effect of cheap imports, but any of our traditional forms of savings rate equation cannot explain the current phenomena. As soon as the savings rate equation rate in the model comes into action, the savings rate jumps up about 7 points, and therefore, this is a variable which needs to be modified in the first few years of the forecast.

4.14 Interest Rate Equations

Interest rates are important to the expenditure side of the model, in that they affect construction and equipment investment, the most cyclical components of expenditure. Previous versions of the model had interest rates in the personal consumption equations as well, and we intend to revisit this specification. Interest rates are also important in the income side of the model, where they affect net interest income, and in various other variables, such as interest paid by federal and state and local governments, and consumer payments of interest to business.

The two most important interest rates are the short- and long-term Treasury bill rates. The short-term rate, or 3-month Treasury bill rate, has the following equation:

\[ rtb = \beta_0 + \beta_1 \text{expinf} + \beta_2 \text{grmbase} + \beta_3 (\text{gnpgap[1]} - 100) + \beta_4 \text{funds} \]

Where:

\( \text{expinf} \) = "Expected" inflation, formed as a three-year average of inflation
\( \text{grmbase} \) = Percent change in the real monetary base
\( \text{gnpgap[1]} - 100 \) = Percentage of actual GNP over potential GNP
\( \text{funds} \) = The share of equipment investment, structures investment plus the federal deficit over GNP

The expected inflation rate illustrates the Fisher effect, where expected increases in inflation tend to get translated into higher nominal interest rates. The base money growth variable is especially effective in predicting interest rates over the last several years. Since 1990, the
monetary base has been growing fairly quickly, whereas M2 has been growing slowly. The GNP gap variable is used to pick up cyclical demand effects on interest rates. The last variable, called *funds*, is intended to capture financial market demand pressure on interest rates. Higher values of this variable indicate a higher demand for finance. The equation for the long-term Treasury bill rate includes the short-term rate, expected inflation, and the *funds* variable. The other interest rate variables are the commercial paper rate, the mortgage rate, and the AAA bond rate. The first two are regressed directly on the long-term rate, and the bond rate is regressed on the long-term rate and the share of profits in GNP.

5. Summary

The LIFT model of the U.S. economy provides a unique framework for analyzing questions where both industry level as well as macroeconomic results are important. Applications that are ideally suited to this modeling framework include: investigations of energy consumption and emissions at the industry level; impacts of free trade agreements on imports and exports by industry; effects on consumption behavior of changing demographics; industry effects of federal tax policy; and price and output responses to a crude oil price increase.

This paper provides an overview of the LIFT model. Several papers and Ph.D. dissertations listed in the references provide more detailed descriptions of particular pieces of the model.
Figure 1. The Flow of the Idlift Model

```c
loop() {
    for(t=godate;t<=stopdate;t++) {
        StartReal(); // Initialize first-pass estimates of some real-side variables
        StartMacro(); // Initialize first-pass estimates of some macrovariables

        Model Loop:
        GetPwt(); // Calculate weighted average foreign/domestic prices ("pwt")
        FDPricing(); // Calculate some final demand prices as pwt passed through bridges
        Savingf(); // Savings rate function
        CrossSect(); // Cross-sectional stage of Pads consumption functions
        ConsumptionEquations.padsf() // PADS consumption equations
        Exports(); // Calculate exports, or take them from BTM

        Investment Output Loop:
        Prepinv(); // Prepare right hand side variables for equipment investment equations
        Invest(); // Equipment investment equations
        Construct(); // Private construction by type
        GovStruct(); // Government construction
        FedGovtExpend(); // Federal government expenditures
        SLGovExpend(); // State & local government expenditures
        Seidel(); // Calculate output, imports and inventory change

        Investment Output Loop Convergence Check
        Productivity(); // Labor productivity
        AvgHours(); // Average hours worked
        Employ(); // Calculate employment, unemployment, unemployment rate

        Price Income Loop:
        PrepareVA(); // Prepare some variables for the price side
        potgnpf(); // Potential GNP, GNP-gap
        Wages(); // Calculate wage rates
        LaborCompensation(); // Calculate labor compensation from hours worked and wage rates
        LaborSplit(); // Split labor compensation into wages, and supplements by type
        Profits(); // Corporate profits
        Propin(); // Proprietors' income
        CorpCCA(); // Corporate capital consumption allowance
        NonCorpCCA(); // Non-corporate capital consumption allowance, including government
        CorporateIVA(); // Corporate inventory valuation adjustment
        NonCorporateIVA(); // Non-corporate inventory valuation adjustment
        BusTransfers(); // Business transfer payments
        NetInterest(); // Net interest payments
        GovtSubsidies(); // Government subsidies
        IndBusTax(); // Indirect business taxes
        ForVA(); // Convert value added by industry to value added by product, and calculate real value added weighted output (revwo).
        #ifdef PSEIDEL // Two alternate methods for calculating prices:
        Pseidel(); // Use seidel method from unit value added
        #else // or
        Markup(); // Use markup equations
        #endif
        RedoVA(); // Recalculate value added to be consistent with markup prices or price fixes
        GetPwt(); // Calculate weighted domestic and import price
        FDPricing(); // Calculate final demand prices
        GovPrices(); // Calculate government prices, and government purchases in current prices

        Price Income Loop Convergence Check
        The Accountant // This consists of several functions written by IdBuild
        FactorIncome() // Factor income to and from rest of world

        Model Loop Convergence Check
        UpdateBuckets(); // Update all buckets used for equipment, structures, etc.
        UpdateCurrentStocks(); // Update current price stocks used to calculate "wear" variables
    }
}
```
References


